Research Article

Data directed root cause analyses of hospital adversities and their proximities

Ramalingam Shanmugam*

School of Health Administration, Texas State University, San Marcos, TX 78666, USA

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*Correspondence:
Dr. Ramalingam Shanmugam,
E-mail: rs25@txstate.edu

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ABSTRACT

Background: In this current era of healthcare reformations, medical professionals, patients, governments, and insurance agencies seek zero tolerance with respect to adverse outcomes. When adversities occur, hospital administrators more often than not perform root cause analysis (RCA) to avoid future reoccurrence. There exist three types of RCA. They are divergent, serial, and convergent root causes. Which adversity type exists in a situation is not medically or intuitively trivial. This article develops a data directed new methodology to characterize the type and interpret it.

Methods: Because tracing root causes of medical adversity is a necessity, pertinent data are collected. Patterns in correlation are examined to check whether it is a divergent or serial type. When it is not either, it is concluded to be convergent. This practice is too elementary to convince professionals. For this purpose, this article innovatively develops a new methodology using inverted correlation matrix and Mahalanobis distances to sort out causes of adversity as serial, divergent, or convergent type. Their proximities are quantifiable due to new expressions in the article. These expressions are not seen in the literature and hence, would benefit practitioners.

Results: A new methodology of this article is illustrated using medical adversities that existed in hospitals during 2006 through 2014 in Indiana state. Data consist of number of surgeries, cases with ulcer acquired in hospital, cases with foreign objects in patient after surgery, cases with wrong part surgery, deaths due to medication error, and disability cases due to fall during hospital treatment. Their correlations ranged from -0.87 to 0.79.

Conclusions: This article has developed expressions to quantify non-equilibrium level in serial and divergent RCA and has demonstrated their use to identify a convergent RCA. The Mahalanobis distance of attained diversities from an ideal scenario is obtained. A formula to make and interpret safety index is developed and demonstrated using adversities that occurred in Indiana State during 2006-2014. These concepts and analytic expressions would enrich the practice of RCA which is a necessity in the current era of healthcare reforms.

Keywords: Serial, Divergent, Convergent root causes

INTRODUCTION

Medical error is adversity and could be fatal at times. What is adversity? It refers any untoward error leading to a patient’s uneasiness, death, life-threat, prolonged hospitalization, disability, birth defect, or health impairment etc. Emergence of electronic health records eases efforts to monitor and fix medical errors if remedial technology is in place. Funds exist to drill for avoiding medical errors (Whitson et al like bio-terrorism Shanmugam).1,2 A medical error is a preventable once its root causes are identified and removed. Medical errors cause 1.5 million human deaths every year in USA. In 2000, the medical costs incurred by preventable drug...
injuries approximated 5887 million (Knudsen et al). Globally, about 142,000 humans died in 2013, an increase from 94,000 in 1990.

Patient’s disclosure is paramount to stop medical errors. About 34 states made a law prohibiting physician’s apology for a medical error in malpractice suit. Not incompetency but rather high risk medical procedures are reasons for adversities (Song et al). A lack of worldwide standards for patient safety is an impediment. Adversities warrant a thorough root cause analyses for sure.

**What is root cause analysis? How does it benefit?**

Root cause analysis (RCA) identifies causal factor(s) that trigger adversities. RCA describes not only what, how, and why adversity happened but also needed remedies to prevent in future (Rooney et al, Zegers et al, and Schafer, 2012). RCA increases patient safety eventually. Causes of medical errors include but are not limited to inexperienced physicians or nurses, non-charted out procedures, older age patients, ambiguous medical communication, language barriers, improper documentation, illegible handwriting, inadequate nurse-to-patient ratios, similarly named medications, patients’ misjudgments, misdiagnosis, administration of wrong drug to wrong patient, giving multiple drugs that badly interact, failure to remove surgical instruments, failure to infuse correct blood type etc. Joint Commission on Accreditation of Healthcare Organizations (JCAHO) requires every medical institution to perform RCA. Guidance on how to conduct RCA is offered by JCAHO. Collecting and analyzing near-miss data is a premium against future adversities. By law, adversities must be reviewed within 45 days of its occurrence by JCAHO to inform patient, to provide support for the staff, to formulate corrective actions, to maintain the confidence of the public, to renew accreditation, to add on in program evaluation among others.

**Data directed new approach to identify rca type**

Every adversity is unique, invariably caused by multiple causes. Data collection about causes is a necessity to understand the scenario. Correlation between variables \( Y_1 \) and \( Y_2 \), partial correlation (Fisher and Kunihiro et al) between two variables by controlling impacts of a third variable \( Y_3 \) on them as indicated by \( \rho_{12} \), \( \rho_{13} \), respectively play crucial role and offer insights. Let their correlation matrix is

\[
\Sigma_{3 \times 3} = \begin{pmatrix}
1 & \rho_{12} & \rho_{13} \\
\rho_{12} & 1 & \rho_{23} \\
\rho_{13} & \rho_{23} & 1
\end{pmatrix}
\]

its inverse matrix is

\[
\Sigma_{3 \times 3}^{-1} = \frac{1}{|D|} \begin{pmatrix}
1 - \rho_{23}^2 & \rho_{13} - \rho_{12} & \rho_{12} - \rho_{13} \\
\rho_{13} - \rho_{12} & 1 - \rho_{13}^2 & \rho_{13} - \rho_{23} \\
\rho_{12} - \rho_{13} & \rho_{13} - \rho_{23} & 1 - \rho_{23}^2
\end{pmatrix}
\]

(2)

In a divergent RCA, two variables are adversities while the third is a cause. In serial or convergent RCA, one variable is adversity, while other two variables are causes.

An example of divergent RCA is the following. When a patient gains body weight (\( Y_1 \)) due to over food consumption, it worsens blood pressure level (\( Y_2 \)) and blood sugar level (\( Y_3 \)). The numerator of their partial correlation, \( \rho_{13|2} = \rho_{12} - \rho_{13}\rho_{23} \) between \( Y_1 \) and \( Y_2 \) is a shift from the level \( \rho_{12} \). With medication or exercise, s/he could force partial correlation to vanish (i.e., \( \rho_{13|2} \rightarrow 0 \)), then the correlation between \( Y_1 \) and \( Y_3 \) remains in its earlier level \( \rho_{12} \). In such divergent RCA, there exists an equilibrium \( \rho_{12} = \rho_{13}\rho_{23} \). Under equilibrium, the inverse matrix is

\[
\Sigma_{3 \times 3, D}^{-1} = \frac{1}{|D_D|} \begin{pmatrix}
1 - \rho_{23}^2 & 0 & -\rho_{12}(1 - \rho_{23}) \\
0 & 1 - \rho_{13}^2 & -\rho_{13}(1 - \rho_{12}) \\
-\rho_{12}(1 - \rho_{23}) & -\rho_{13}(1 - \rho_{12}) & 1 - \rho_{12}^2
\end{pmatrix}
\]

(3)

with determinant

\[
D_D = (1 + \rho_{12}^2) - (\rho_{13}^2 + \rho_{23}^2)
\]

(4)

Does a serial RCA undergo change similarly? Consider the following example. When an unusual heavy rain level (\( Y_1 \)) due to a stationary thunderstorm like Katrina in New Orleans occurs in a town, the available number (\( Y_2 \)) of trained medical staff (including doctors, nurses, and support personnel etc.) in a hospital reduces. Consequently, the number (\( Y_3 \)) of intensive care patients to be evacuated to nearby hospital sites that have all needed electric power to support medical devices. Such domino effect of three variables exist. Hence, the scenario gets the name serial RCA. Remedies include having an emergency plan to airlift patients or healthcare
administrators, and/or an emergency standby electric power generation. These remedies impact \( Y_2 \). The numerator of the partial correlation, \( \rho_{13|2} = \rho_{13} - \rho_{12}\rho_{23} \) between \( Y_1 \) and \( Y_3 \) is a shift from its initial correlation \( \rho_{13} \). If the shift is negligible (that is equivalent to \( \rho_{13|2} \to 0 \)), there exists an equilibrium \( \rho_{13} = \rho_{12}\rho_{23} \) in serial RCA. Under equilibrium, the inverse matrix is

\[
\Sigma^{-1}_{3,3} = \frac{1}{[D_{\Sigma}]} \begin{pmatrix}
(1 - \rho_{13}^2) & -\rho_{12}(1 - \rho_{23}^2) & 0 \\
-\rho_{12}(1 - \rho_{23}^2) & (1 - \rho_{13}^2) & -\rho_{23}(1 - \rho_{13}^2) \\
0 & -\rho_{23}(1 - \rho_{13}^2) & (1 - \rho_{13}^2)
\end{pmatrix}
\]  

(5)

With determinant

\[ D_{\Sigma} = (1 + \rho_{13}^2) - (\rho_{12}^2 + \rho_{23}^2) \]  

(6)

An example of convergent RCA is the following. When an operating surgeon has a far exceeding fatigue level (\( Y_1 \)) due to moonlighting under compulsion and the patient on operating theater has an unclear notes level (\( Y_2 \)) in medical dossier, an adversity of transplanting a wrong kidney occurs (\( Y_3 \)). Here, \( Y_3 \) and \( Y_2 \) are causes contributing to a surgical error \( Y_1 \) in a convergent manner. The numerator of the partial correlation, \( \rho_{23|1} = \rho_{23} - \rho_{13}\rho_{12} \) between \( Y_1 \) and \( Y_2 \) shifts from an initial level \( \rho_{23} \). There is no possible remedial action to enforce the partial correlation to vanish (i.e., \( \rho_{23|1} \neq 0 \)). No equilibrium exists in convergent RCA. If it is not serial RCA nor divergent RCA, then it is convergent RCA (Alemi). Such a classification is too elementary to convince professionals. This article devises a better classification methodology. For this purpose, let

\[ 0 \leq \pi_D = \frac{\rho_{13}\rho_{23}}{\rho_{12}} < 1 \quad \text{and} \quad 0 \leq \pi_S = \frac{\rho_{13}\rho_{23}}{\rho_{12}} < 1 \]

describe non-negative equilibrium levels. When \( \pi_D = 1 \), it is divergent RCA. When \( \pi_S = 1 \), it is serial RCA. An imbalance \( D_C = 1 - \sqrt{\pi_D\pi_S} + \rho_{13}^2(1 - \pi_S) \{1 - \varrho_{23|1/D} \} \) (7)

where

\[ \varrho_{23|1/D} = \frac{\text{Imbalance}_{s}}{\text{Imbalance}_{d}} \]  

(8)

is an imbalance ratio for RCA to be serial to be divergent. The inverted correlation matrix is

\[
\Sigma^{-1}_{3,3} = \frac{1}{[D_{\Sigma}]} \begin{pmatrix}
1 - \pi_D^2 & -\pi_D \rho_{13}(1 - \pi_S) & -\pi_D \rho_{13}(1 - \pi_S) \\
-\pi_D \rho_{13}(1 - \pi_S) & 1 - \pi_S^2 & -\rho_{13}(1 - \pi_S) \\
-\pi_D \rho_{13}(1 - \pi_S) & -\rho_{13}(1 - \pi_S) & 1 - \rho_{13}^2
\end{pmatrix}
\]  

(9)

In medical applications where it is not clear how an adversity drifts from divergent or serial, the imbalance ratio (8) assesses their proximities. When \( \rho_{13} \) is much larger than \( \rho_{12}\rho_{23} \), the imbalance \( \pi_S \) is negligible, the determinant (7) reduces to \( D_C \to 1 + \rho_{13}^2 \) with no trouble to invert \( \Sigma_{3,3} \). On the contrary, when \( \rho_{12} \) is much larger than \( \rho_{13}\rho_{23} \), the imbalance \( \pi_D \) is negligible, the determinant (7) becomes infinity (that is, \( D_C \to \infty \)) with inability to invert \( \Sigma_{3,3} \). Consequently, the RCA falls in a “black hole” beyond comprehension. Data might exist out there of this scenario in real life and data analyst assisting decision makers should be beware of data pitfalls. This is a take-home lesson from this article.

If \( \pi_D \to 0 \) and \( \pi_S \to 0 \), a possibility, then the determinant approaches one (that is, \( D_C \to 1 \)) and hence, inversion of \( \Sigma_{3,3} \) is possible. What is so big deal to invert \( \Sigma_{3,3}^{-1} \)? It helps to quantify the Mahalanobis gap between two red points in space generated by correlated

RCA. After simplifications for a convergent RCA, the determinant (1) is

\[ D_C = 1 - \sqrt{\pi_D\pi_S} + \rho_{13}^2(1 - \pi_S) \{1 - \varrho_{23|1/D} \} \]  

(7)

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data variables. Absolute gap is irrelevant, when data shape is unevenly stretched in one direction than the other direction as in Figure 1. Mahalanobis distance becomes Euclidean distance in orthogonal space. The Mahalanobis distance

\[
MD_{y_1,y_2} = (y_1 - y_2)' \Sigma^{-1} (y_1 - y_2)
\]

is dispersion gravitated gap between two red colored locations \(y_1\) and \(y_2\) in elliptically shaped data space generated by correlated variables. In a safe situation with no cause and no adversity, the data location is \(y = 0\). A data location with attained causes and adversities is \(\Sigma^{-1} \Sigma y\) distance away from the safety location. In a divergent RCA, this distance is \(\sum D \Sigma^{-1} D\), according to inverted correlation matrix (3) and determinant (4). In a serial RCA, this distance is \(\sum S \Sigma^{-1} S\), according to inverted correlation matrix (5) and determinant (6).

Illustrations using hospital adversities in Indiana state

To illustrate the new approach in this article, data about medical adversities that occurred in hospitals during 2006 through 2014 in Indiana state are considered. The variables are number of surgeries, number of cases acquiring ulcer in hospital, with foreign object in patient after surgery, with wrong part surgery, with disability due to medication error (Hover et al and Knudsen et al) and with disability due to fall as in Table 1.

![Figure 1: Data space and Mahalanobis distance.](image)

<table>
<thead>
<tr>
<th>Year</th>
<th>(Y_1) = # surgeries</th>
<th>(Y_2) = # cases got ulcer due to hospital infection</th>
<th>(Y_3) = # cases with foreign object in patient after surgery</th>
<th>(Y_4) = # cases with wrong part surgery</th>
<th>(Y_5) = # cases with adversity due to medication error</th>
<th>(Y_6) = # cases with adversity due to patients falling</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>39</td>
<td>26</td>
<td>23</td>
<td>11</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>2007</td>
<td>49</td>
<td>27</td>
<td>24</td>
<td>23</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>2008</td>
<td>48</td>
<td>33</td>
<td>30</td>
<td>16</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>2009</td>
<td>51</td>
<td>22</td>
<td>29</td>
<td>17</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2010</td>
<td>50</td>
<td>34</td>
<td>33</td>
<td>14</td>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>2011</td>
<td>40</td>
<td>41</td>
<td>17</td>
<td>18</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>2012</td>
<td>41</td>
<td>30</td>
<td>19</td>
<td>15</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>2013</td>
<td>48</td>
<td>45</td>
<td>27</td>
<td>18</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>2014</td>
<td>52</td>
<td>44</td>
<td>27</td>
<td>21</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>(\bar{Y})</td>
<td>46.44</td>
<td>33.56</td>
<td>25.44</td>
<td>17</td>
<td>3.44</td>
<td>10</td>
</tr>
<tr>
<td>(s^2)</td>
<td>25.27</td>
<td>67.78</td>
<td>27.03</td>
<td>13</td>
<td>9.53</td>
<td>17.75</td>
</tr>
</tbody>
</table>
Table 2: Correlation $\rho_{ij}$, $i, j = 1, 2, 3, 4, 5, 6, 7, 8, 9$ among the variables in Table 9.

<table>
<thead>
<tr>
<th></th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
<th>$Y_5$</th>
<th>$Y_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_2$</td>
<td>0.11</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_3$</td>
<td>0.79**</td>
<td>-0.05</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_4$</td>
<td>0.52</td>
<td>0.31</td>
<td>-0.06</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_5$</td>
<td>0.03</td>
<td>-0.36</td>
<td>-0.02</td>
<td>0.27</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$Y_6$</td>
<td>0.08</td>
<td>0.46</td>
<td>0.108</td>
<td>-0.14</td>
<td>-0.87**</td>
<td>1</td>
</tr>
</tbody>
</table>

**refers less than 0.001 with determinant $D = 0.003$ of correlation matrix $\Sigma_{3x3}$

Figure 2: Proximity among the six variables of data in Table 1.

The variables are correlated (see Table 2). The inverted correlation matrix is

$$\Sigma^{-1}_{6x6} = \begin{pmatrix} 41.5 & 4.2 & -34.4 & -27.3 & 9.9 & 3.4 \\ 2 & -3.3 & -3.3 & 1 & -0.5 \\ 29.5 & 22.6 & -8.6 & -3.3 \\ 19.3 & -7.3 & 2.4 \\ 6.9 & 4.6 & 5 \end{pmatrix}$$

with its determinant $D = 0.003$, a small quantity indicating near singularity. It means root causes are too mixed to visualize clear classifications.

Among the six variables, only $Y_1$ is non adversity, while other five are adversities, should be ideally zero. Such a scenario is named safety location in data space. Its location is $\delta_{\text{service}} = \bar{Y}_{\text{service}} \Sigma^{-1}_{6x6} \bar{Y}_{\text{service}}$ distance from origin $O$, where $\bar{Y}_{\text{service}} = (\bar{Y}_1, 0, 0, 0, 0, 0)$ is hospital’s service level. The origin cannot be a safety index because

in it, the hospital should provide a zero surgery. Data directed index of safety level is needed and it is

$$SI = \text{SafetyIndex} = 1/(1 + \delta_{\text{service}}).$$

$SI$ is best near origin and reduces as $\delta_{\text{service}}$ increases. For data in Table 1, $\bar{Y}_{\text{service}} = (46, 0, 0, 0, 0, 0)$ and $\delta_{\text{service}} = 87814$ using (10). With extremely high service level, the safety index is almost zero (that is, $SI \approx 0$). With attained adversities (see Table 1), $\bar{Y}_{\text{attained}} \approx (46, 33, 25, 17, 3, 10)$ and

$$\delta_{\text{attained}} = \frac{(\bar{Y}_{\text{attained}} - \bar{Y}_{\text{service}}) \Sigma^{-1}_{6x6} (\bar{Y}_{\text{attained}} - \bar{Y}_{\text{service}})}{\text{distance}} = 32461$$

further from $\delta_{\text{service}} = 87814$. The safety index is zero (that is, $SI \approx 0$). Detailed RCA is definitely needed.

Let us begin with a multivariate principal component analysis. Three orthogonal principal components (PC) explain 90.24% of the data variations. See Figure 2. The first PC selects $Y_2$, $Y_5$, and $Y_6$ as important variables. The second PC selects $Y_4$, $Y_1$, and $Y_3$ as next important variables. The third PC selects $Y_2$, $Y_3$, and $Y_4$ as last important variables. This is what data directed RCA.

$$\Sigma_{3x3} = \begin{pmatrix} 1 & \rho_{Y_2Y_5} & \rho_{Y_2Y_6} \\ \rho_{Y_2Y_5} & 1 & \rho_{Y_5Y_6} \\ \rho_{Y_2Y_6} & \rho_{Y_5Y_6} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -0.36 & 0.47 \\ -0.36 & 1 & -0.87 \\ 0.47 & -0.87 & 1 \end{pmatrix}$$

with $\rho_{ij} = \rho_{ij}$ for $i, j = 1, 2, 3$.
**RCA-1: Getting ulcer in hospitalization (Y2), medication error (Y5), versus falling (Y6)**

This RCA could be divergent or serial. Their correlation matrix is with determinant is $D = 0.2$. The inverted correlation matrix is

$$
\Sigma^{-1}_{3x3} =
\begin{pmatrix}
1.3 & -0.2 & -0.8 \\
-0.2 & 4 & 3.6 \\
-0.8 & 3.6 & 4.5 \\
\end{pmatrix}
$$

Their proximities are displayed in Figure 3.

The desired is $\bar{Y}_{desired} = (0,0,0)$ which would have resulted in $\delta_{desired} = \bar{Y}_{desired} \Sigma^{-1}_{3x3} \bar{Y}_{desired} = 0$ and $SI = 1$.

But, $\bar{Y}_{attained} \approx (33,3,10)$. The attained location is $\delta_{attained} = \bar{Y}_{attained} \Sigma^{-1}_{3x3} \bar{Y}_{attained} = 1550$ distance from desired location $\delta_{desired}$. The attained safety index is $SI = 1/(1 + \delta_{attained}) = 0.0006$.

**Figure 3: Proximities of Y2, Y5, and Y6.**

Is Y2 a cause? Are Y5 and Y6 adversities? If so, is it a divergent RCA? But, $\rho_{Y2Y5} \neq \rho_{Y2Y6}$ according to (11). Hence, it is not divergent, with $\pi_D = 0.19$, not required $\pi_D = 1$. The imbalance is 23/100 for divergent versus non-divergent. Y5 is more likely to be causal than Y2.

Is RCA-1 serial? If so, which among six possibilities did happen? One possibility is Y2 as root cause, Y5 as direct cause for adversity Y6, as indicated by notation $Y_2 \rightarrow Y_5 \rightarrow Y_6$. According to (11), $\rho_{Y2Y6} \neq \rho_{Y2Y5} \rho_{Y5Y6}$. They are in that serial. The equilibrium level is $\pi_5 = 0.66$. The imbalance is 50/100 in favor of serial.

Likewise, the imbalance for serial types $Y_5 \rightarrow Y_2 \rightarrow Y_6$, $Y_2 \rightarrow Y_6 \rightarrow Y_5$, $Y_5 \rightarrow Y_6 \rightarrow Y_2$, $Y_2 \rightarrow Y_6 \rightarrow Y_5 \rightarrow Y_2 \rightarrow Y_6$, $Y_2 \rightarrow Y_6 \rightarrow Y_5 \rightarrow Y_6 \rightarrow Y_2 \rightarrow Y_6$, $Y_2 \rightarrow Y_6 \rightarrow Y_5 \rightarrow Y_6 \rightarrow Y_2 \rightarrow Y_6$ are respectively 20/100, 840/100, 840/100, 20/100, 2. It appears the domino effects are both ways (that is, $Y_5 \rightarrow Y_6 \rightarrow Y_5$ or $Y_5 \rightarrow Y_5 \rightarrow Y_2$). Either Y2 or Y5 is root cause with Y6 is direct cause for Y1 or Y2. However, the RCA-1 is serial. Reality is sometimes not that crystal clear to conclude.

**RCA-2: Numbers of surgeries (Y1), foreign objects in patient after surgery (Y3), versus wrong part surgeries (Y4)**

The RCA-2 consists of Y1, Y3, and Y4. Their correlation matrix is

$$
\Sigma_{3x3} =
\begin{pmatrix}
1 & \rho_{Y1Y3} & \rho_{Y1Y4} \\
\rho_{Y1Y3} & 1 & \rho_{Y3Y4} \\
\rho_{Y1Y4} & \rho_{Y3Y4} & 1 \\
\end{pmatrix}
$$

with determinant is $D = 0.05$. The inverted correlation matrix is

$$
\Sigma^{-1}_{3x3} =
\begin{pmatrix}
20.9 & -17.2 & -12 \\
-17.2 & 15.2 & 9.9 \\
-12 & 9.9 & 7.9 \\
\end{pmatrix}
$$

(13)

With
Figure 4: Proximities of Y1, Y3, and Y4.

Were there too many surgeries? Is Y1 a root cause of adversities Y3 and Y4? With no adversities, desired \(\bar{y}_{\text{desired}} = (46,0,0)\) which would have resulted in \\
\[ \delta_{\text{desired}} = \bar{y}_{\text{desired}}^{-1} \cdot \bar{y}_{\text{desired}} = 961.4 \]
and \\
\[ SI = 1/(1 + \delta_{\text{desired}}) = 0.001. \]

But \(\bar{y}_{\text{attained}} \approx (46,25,17)\). The attained location is \\
\[ \delta_{\text{attained}} = \bar{y}_{\text{attained}}^{-1} \cdot \bar{y}_{\text{attained}} = 24913.1 \] distance from
desired location \(\delta_{\text{desired}}\). The attained index of safety is \\
\[ SI = 1/(1 + \delta_{\text{attained}}) = 4.01379E-05. \]

Is RCA-2 a divergent? Because, \(\rho_{Y3Y4} \neq \rho_{Y1Y2} \rho_{Y3Y4}\) according to (13), it is not divergent, with \(\pi_{D} = 4\), not required \(\pi_{D} = 1\). The imbalance is 133/100 in favor of divergent.

Is RCA-2 serial? If so, there are two possibilities. Y1 as root cause, Y3 as direct cause for adversity Y4, as indicated by notation \(Y_1 \rightarrow Y_3 \rightarrow Y_4\). According to (13), \\
\[ \rho_{Y3Y4} \approx \rho_{Y1Y2} \rho_{Y3Y4}\] They are in serial. The equilibrium level is \(\pi_{S} = 0.8\). Hence, the imbalance are 110/100 for serial \(Y_1 \rightarrow Y_3 \rightarrow Y_4\).

Or, is it serial \(Y_1 \rightarrow Y_4 \rightarrow Y_3\)? According to (13), \\
\[ \rho_{Y3Y4} \neq \rho_{Y1Y2} \rho_{Y3Y4}\] They are not in serial. The equilibrium level is \(\pi_{S} = 0.1\), not required \(\pi_{S} = 1\).

Hence, the imbalance is 90/100 for serial \(Y_1 \rightarrow Y_4 \rightarrow Y_3\).

In conclusion, RCA-2 is divergent.

RCA-3: Numbers of ulcer acquired in hospital (Y2),
foreign objects in patient after surgery (Y3), versus
wrong part surgery (Y4)

RCA-3 makes an assessment of Y2, Y3, and Y4 as a set. Their correlation matrix is

\[
\Sigma_{3x3} = \begin{pmatrix}
1 & \rho_{Y2Y3} & \rho_{Y2Y4} \\
\rho_{Y2Y3} & 1 & \rho_{Y3Y4} \\
\rho_{Y2Y4} & \rho_{Y3Y4} & 1
\end{pmatrix}
\]

with determinant is \(D = 0.9\). Their inverted correlation matrix is

\[
\Sigma_{3x3}^{-1} = \begin{pmatrix}
1.1 & 0.4 & -0.3 \\
0.4 & 1 & 0.05 \\
-0.3 & 0.05 & 1.1
\end{pmatrix}
\]

With

Figure 5: Proximities of Y2, Y3, and Y4.

All three Y2, Y3, and Y4 are adversities. What is desired is \(\bar{y}_{\text{desired}} = (0,0,0)\) which would have resulted in \\
\[ \delta_{\text{desired}} = \bar{y}_{\text{desired}}^{-1} \cdot \bar{y}_{\text{desired}} = 0 \]
and \\
\[ SI = 1/(1 + \delta_{\text{desired}}) = 1. \] But, \(\bar{y}_{\text{attained}} \approx (33,25,17)\).

The attained location is \\
\[ \delta_{\text{attained}} = \bar{y}_{\text{attained}}^{-1} \cdot \bar{y}_{\text{attained}} = 2506.7 \] distance from
desired location $\delta_{\text{desired}}$. The attained safety index is $SI = 1/(1 + \delta_{\text{attained}}) = 0.0003$.

Is RCA-3 divergent? Is $Y_2$ a cause? Are $Y_3$ and $Y_4$ adversities? But, $\rho_{Y_3Y_4} \neq \rho_{Y_2Y_3}$, according to (15).

Hence, it is not divergent, with $\pi_D = 0.3$, not required $\pi_D = 1$. The imbalance is 4/100 in favor of divergent.

Is $Y_3$ a cause? Are $Y_2$ and $Y_4$ adversities? Is RCA-3 divergent? Clearly, $\rho_{Y_2Y_4} \neq \rho_{Y_2Y_3}$, according to (15).

Hence, it is not divergent, with $\pi_D = 0.03$, not required $\pi_D = 1$. The imbalance is 3/100 in favor of divergent. If at all RCA-3 is divergent (which is unlikely), then $Y_2$ is more likely than $Y_3$ is the cause.

Is RCA-3 serial? One possibility is $Y_2$ as root cause, $Y_3$ as direct cause for adversity $Y_4$, as indicated by notation $Y_2 \rightarrow Y_3 \rightarrow Y_4$. According to (15), $\rho_{Y_3Y_4} \neq \rho_{Y_2Y_3}$.

The equilibrium level is $\pi_3 = 0.03$. The imbalance is 3/100 to be serial. Computing likewise, the imbalance in favor of serial $Y_3 \rightarrow Y_2 \rightarrow Y_4$ is 43/100. What are the imbalance for $Y_4$ to be root cause for a serial? For serial $Y_4 \rightarrow Y_2 \rightarrow Y_3$, the imbalance is 43/100. For serial $Y_4 \rightarrow Y_2 \rightarrow Y_3$, the imbalance is 3/100. Hence, RCA-3 is a serial type $Y_3 \rightarrow Y_2 \rightarrow Y_4$ or in an opposite serial direction $Y_4 \rightarrow Y_2 \rightarrow Y_3$. That is, leaving foreign objects in patient’s body was the root cause to getting ulcer during hospital treatment by the patient who later found out of wrong surgery. Or, root cause was performing surgery on a wrong part leading to a direct cause of getting ulcer in hospital stay to eventually notice foreign objects in patient’s body. Medical reality is vague at times to trace out by numbers alone.

**Recommendations**

Undoubtedly, RCA is a time consuming process but is certainly valuable to learn lessons from past mistakes with a determination of not allowing them to happen ever again. If a situation is a serial type, ought to find out its root causes to fix them first before wasting resources or time to eliminate direct causes for an intention not to have any adversity. If a situation is a divergent type, fixing of several adversities can be accomplished by just finding and eliminating its root cause to avoid adversities. If a situation is neither serial nor divergent type, then it is convergent type. For a convergent situation, hospital administrators have to find out several causes to fix an adversity. Evidently, dealing with convergent situation is tedious but is a necessity to attain a zero tolerance in medical care of patients. This article has constructed and demonstrated data directed root cause analysis.

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